Flux breaking and natural Standard Model structure in F-theory

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Based on 2112.03947 and 2207.xxxxx with Washington Taylor

Goal: SM structure[1] from rigid GUT

- Seemingly the most **natural** way to realize SM structure in 4D F-theory models [Taylor, Wang]
- Moduli independent, relying on local geometry
- E_7 GUT but with chiral matter
- No chiral exotics with intermediate $\mathrm{SU}(5)$
- Linear Diophantine chiral indices, naturally small, e.g.

$$\chi = 5n_1 - 7n_2 + 3n_3 - 10n_4 + \cdots, n_i \in \mathbb{Z}$$

- [1]The gauge group and chiral matter spectrum

Vertical and remainder flux breaking

Vertical flux	Remainder flux
$G_4^{vert} \in \mathrm{span}([D_i] \wedge [D_\alpha])$	$G_4^{rem} \in \mathrm{span}\left([D_i\vert_{C_{rem}}]\right)$
Non-abelian part: breaks to commutant of flux direction	
Breaks some $\mathrm{U}(1)$'s by Stückelberg mechanism	Preserves all $\mathrm{U}(1)$'s
Induces chiral matter	Does not induce chiral matter
Nontrivial primitivity ($J \wedge G_4 = 0$) constraints	Primitivity satisfied

[Braun, Watari]

Notations:
D_i - exceptional Cartan divisors
D_α - (pullback of) base divisors
C_{rem} - curve on gauge divisor Σ, homologically trivial on base B

Flux constraints

- Flux quantization: $G_4 + \frac{1}{2}c_2(\hat{Y}) \in H^4(\hat{Y}, \mathbb{Z})$ [Witten] → assume integer ϕ
- Tadpole: $N_{D3} = \frac{\chi(\hat{Y})}{24} - \frac{1}{2}\int_{\hat{Y}} G_4 \wedge G_4$ [SVW, DM, DRS] → natural to have small ϕ
- **Primitivity**: $J \wedge G_4 = 0$ [Becker, Becker; GVW] → subtle!
- Poincaré invariance: $\Theta_{0\alpha} = \Theta_{\alpha\beta} = 0$

$$G_4^{vert} = \phi_{IJ}[D_I] \wedge [D_J], \Theta_{IJ} = \int_{\hat{Y}} G_4 \wedge [D_I] \wedge [D_J]$$

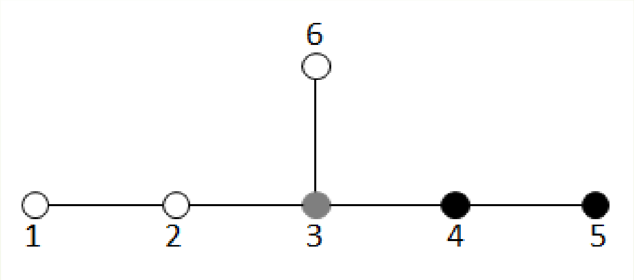
Our recipe

E_7 or E_6 $\xrightarrow{\text{vertical}}$ $\mathrm{SU}(5)$ $\xrightarrow{\text{remainder}}$ G_{SM}

- Pure vertical breaking leads to exotic $\mathrm{U}(1)$, need "hypercharge flux"

[Beasley, Heckman, Vafa; Donagi, Wijnhol; Blumenhagen, Grimm, Jurke, Weigand; Marsano, Saulina, Schafer-Nameki; Grimm, Krause, Weigand]

- Challenge: rigid E_7/E_6 + C_{rem} + primitivity + $\chi = 3$
- Requires **non-toric** base



Explicit E_6 example

- Base as toric hypersurface [Braun, Collinucci, Valandro]
- **Step 1**: A threefold A with $h^{1,1} > 2$ $(\Delta(\mathrm{rank}))$
- E.g. $A = \mathbb{P}^1 \times \mathbb{F}_1$
- On $\mathbb{F}_1$, let $s =$ section, $f =$ fiber
- On A, let $\sigma = \mathbb{F}_1$ section, $S, F =$ fibers along s, f
- **Step 2**: A fourfold X as $\mathbb{P}^1$-bundle over A with normal bundle $N_A = -a\sigma$
- Let $\sigma_A = A$ section, $F_\sigma, F_S, F_F =$ fibers along σ, S, F

Explicit E_6 example

- **Step 3**: Base B as ample divisor in X with class

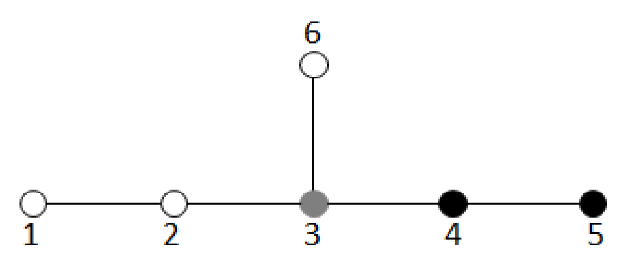
$$B = \sigma_A + (a+1)F_\sigma + 2F_S + 3F_F$$

- $H^{1,1}(X,\mathbb{Z}) \cong H^{1,1}(B,\mathbb{Z})$ by Lefschetz's hyperplane theorem
- Effective $-K_B = B \cdot (\sigma_A + F_\sigma)$
- **Step 4**: Rigid gauge divisor $\Sigma = B \cdot \sigma_A$

$$-K_\Sigma = B \cdot \sigma_A \cdot F_\sigma, N_\Sigma = -aB \cdot \sigma_A \cdot F_\sigma$$

- Choose $a = 3$ for rigid E_6
- Can show $h^{1,1}(\Sigma) = 10 > h^{1,1}(B) = 4 \rightarrow$ existence of C_{rem}

Explicit E_6 example



- **Step 5**: Vertical flux breaking $E_6 \rightarrow \mathrm{SU}(5)$, giving chiral $\mathbf{5}, \mathbf{10}$

$$\Theta_{1\alpha} = \Theta_{2\alpha} = \Theta_{3\alpha} = \Theta_{6\alpha} = 0 \rightarrow (\phi_{1\alpha}, \phi_{2\alpha}, \phi_{3\alpha}, \phi_{4\alpha}, \phi_{6\alpha}) = (2,4,6,5,3) n_\alpha$$

$$\chi = \Sigma \cdot (6K_B + 5\Sigma) \cdot D_\alpha n_\alpha = -3(n_S + 2n_F)$$

[Donagi, Wijnholt; Beasley, Heckman, Vafa; Braun, Collinucci, Valandro; Marsano, Schafer-Nameki; Krause, Mayrhofer, Weigand; Grimm, Hayashi]

- (No chiral matter before flux breaking due to geometry)
- **Step 6**: Remainder flux breaking $\mathrm{SU}(5) \rightarrow \mathrm{SU}(3) \times \mathrm{SU}(2) \times \mathrm{U}(1)/\mathbb{Z}_6$

$$G_4^{rem} = \left[(2D_1 + 4D_2 + 6D_3 + 3D_6)\big|_{C_{rem}} \right]$$

Explicit E_6 example

- **Step 7**: Choose n_α to satisfy primitivity

$$[J_B] = B \cdot (t_1 F_F + t_2(F_S + F_F) + t_3 F_\sigma + t_4(\sigma_A + 3F_\sigma))$$

- Inside Kähler cone of X (hence inside that of B), $t_a > 0$
- Primitivity $J \wedge G_4 = 0$:

$$t_1(2n_\sigma + n_S) + t_2(3n_\sigma + n_F) + t_3(n_S + 2n_F) = 0$$

- Minimal flux configuration: (recall scarcity from tadpole)

$$(n_\sigma, n_S, n_F) = (1, -1, 0) \rightarrow \boldsymbol{\chi = 3}$$

- Stabilizes some Kähler moduli within the Kähler cone

Future directions

- Higgs/vector-like exotics
- Yukawa/proton decay
- Moduli stabilization
- (Heterotic) dual theories
- Remnants in low energy
- Distributions in the landscape
- Exotic models